The Free Particle in Quantum Mechanics

Phys 401

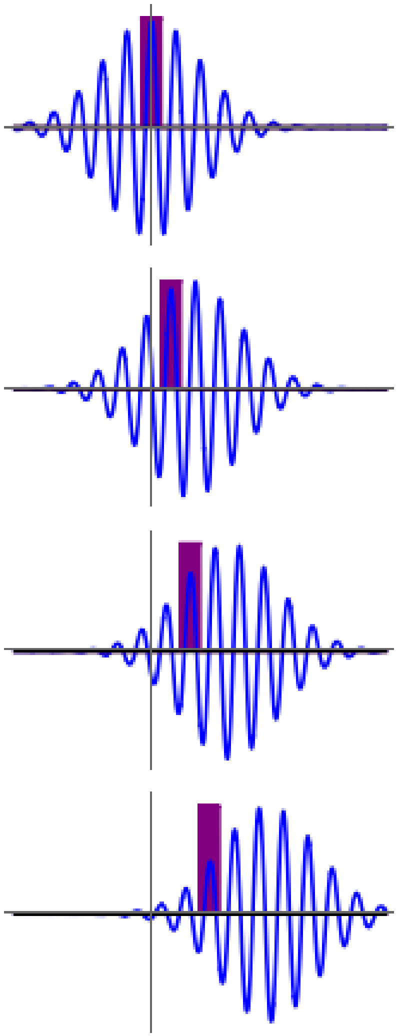
$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi(k) e^{i(kx - \frac{\hbar k^2}{2m} t)}\, dk$$

$$\Psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \phi(k) e^{ikx}\, dk$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(k) e^{ikx}\, dk \iff F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx}\, dx$$

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x,0) e^{-ikx}\, dx$$

Phase and Group Velocity of a Quantum Free Particle Wave Packet



Propagation of a wave packet, with the motion of a single peak shaded in purple. The peaks move at the phase velocity while the overall packet moves at the group velocity.

$$v_p = \frac{\omega}{k} = \frac{\hbar k}{2m} = \frac{p}{2m}.$$

$$v_g = \nabla\omega(\mathbf{k}) = \frac{\hbar \mathbf{k}}{m} = \frac{\mathbf{p}}{m},$$

https://en.wikipedia.org/wiki/Free_particle

An Example Free Particle Problem

$$\Psi(x,0) = \begin{cases} A, & -a < x < a \\ 0, & otherwise \end{cases}$$

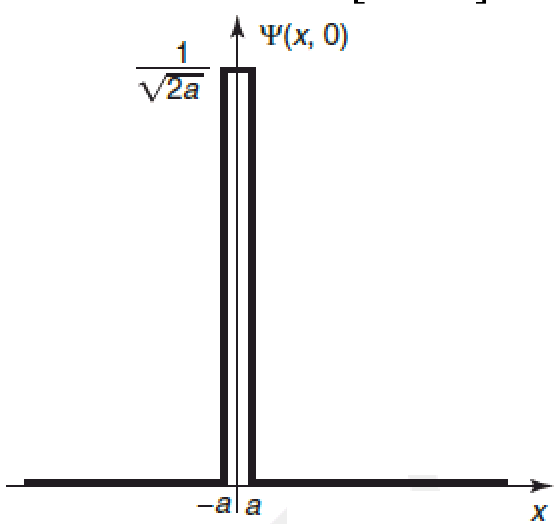
Where A and a are positive constants

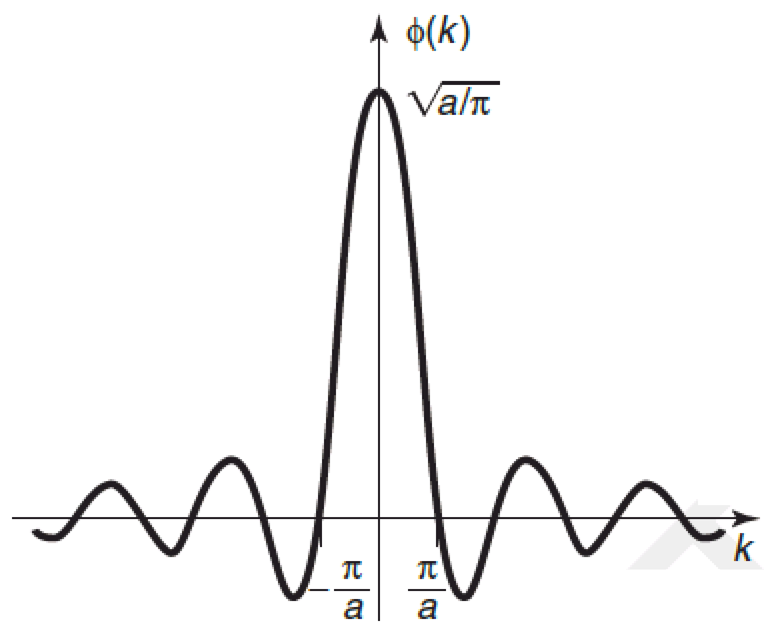
Find $\Psi(x,t)$

Normalization gives $A = 1/\sqrt{2a}$

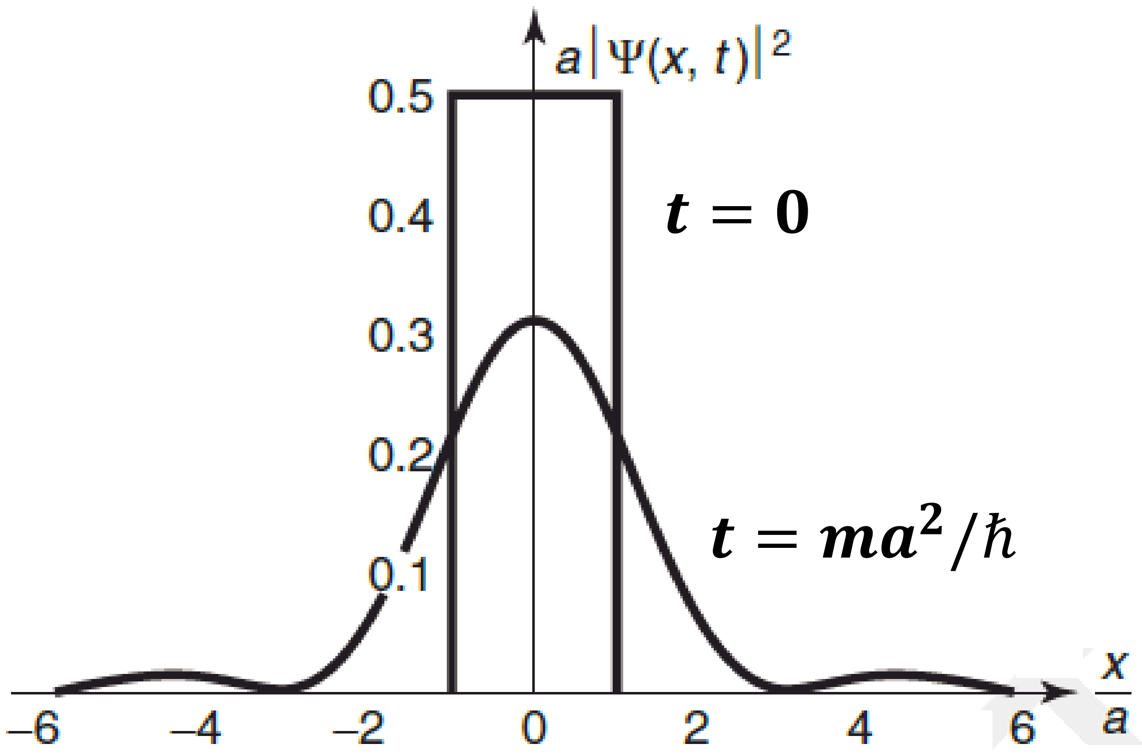
$$\phi(k) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2a}}\int_{-a}^{a} e^{-ikx}dx = \frac{1}{\sqrt{\pi a}}\frac{\sin(ka)}{k}$$

A particle initially confined to the interval $x \in [-a, a]$





$$\Psi(x,t) = \frac{1}{\pi\sqrt{2a}} \int_{-\infty}^{\infty} \frac{\sin(ka)}{k} e^{i(kx - \frac{\hbar k^2}{2m} t)} \, dk$$



Griffiths Problem 2.20